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Estimation of Unsteady Loading for Sting Mounted Wind Tunnel Models

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Measurement of static loads is conventionally performed using a linear combination of strain gage measurements. For the case of a loading environment with energy content in the same frequency band as structural modes, measurements can be significantly corrupted. In this development, a method for accurately estimating the spectral content of an applied loading is presented. This is accomplished by performing system identification on the structural dynamic system using a prescribed forcing function. A linear filter is developed based on the identified plant that converts measured strain and acceleration data to an estimate of applied loads. Analytical and experimental results are presented for a simple structural dynamic system subjected to a prescribed forcing function.

I. Introduction

Challenges to dynamic store load measurement include vibration of the support system and limitations of the balance and data acquisition system. The Phase I effort will include characterization of these issues as well as any additional issues identified during the investigation.

Vibration of a store load measurement system can include excitation of the store model, balance, sting, or even the wind tunnel support structure. If such vibrations are in the same frequency band as the unsteady aerodynamic forces to be measured, they will seriously corrupt the results. Investigations of weapons bays at model size have identified aerodynamic Rossiter modes at frequencies on the order of 1 kHz[1]. Inspection of existing sting configurations (Figure 1) indicates that many structural dynamic modes are present below this frequency.

In addition to the issue of undesired structural vibrations, limitations may be introduced by the balance and data acquisition system. Piezoelectric load cells exist that are capable of accurately measuring forces at the frequencies required for characterizing weapons bay unsteady aerodynamics. The use of such dynamic load cells could address the problem of resonance of the balance itself. Additionally, data acquisition can be performed in a manner to capture and preserve the unsteady aerodynamic content of interest. This includes selecting a sampling rate such that the Nyquist frequency is well above the frequency range of interest for unsteady load content. Anti-aliasing filters can be used and set with cutoff filters between the range of interest and the Nyquist frequency. Once these steps are taken, provisions can be made to store large time domain data records or to convert measured data to the frequency domain as part of the testing procedure.

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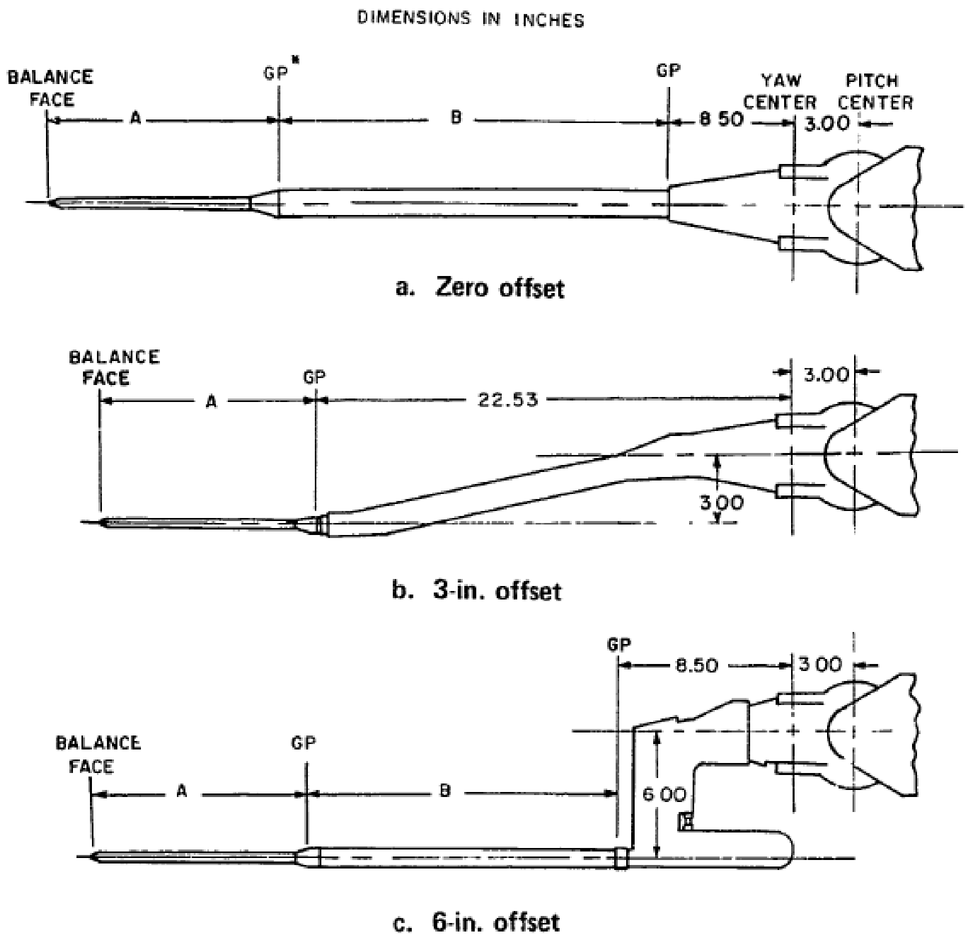


Figure 1: Existing CTS Sting Configurations[2]

II. Technical Approach

A. Overview

A measurement system is designed for unsteady load estimation. The system dynamics between the inputs (aerodynamic forces on the store model) and outputs (strain and acceleration measurements) are identified using output data generated with a known input signal. Once these system dynamics are determined, the input data can be calculated for measured output data series. Figure 2 shows an illustration of the process of measurement system design.

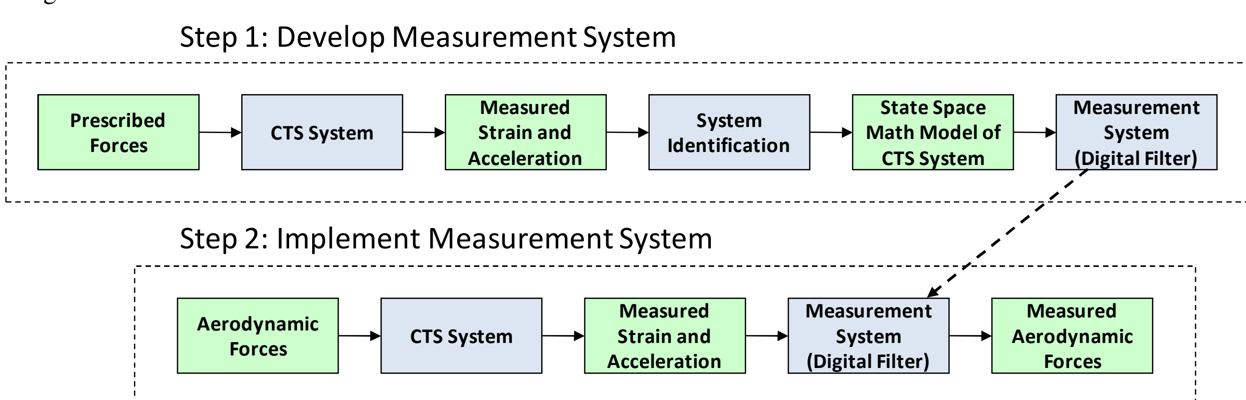


Figure 2: Illustration of Measurement System Design Process.

B. System Identification

Multiple methods exist for identifying linear systems from input and output data. These include the subspace method[3,4], PolyMAX[5], and the Eigensystem Realization Algorithm[6]. This development utilizes the subspace method to identify a state space model of the form:

$$\dot{x} = Ax + Bu \quad . \tag{1}$$

$$y = Cx + Du \quad . \tag{2}$$

where *u* are the inputs (loads), *y* are the outputs (measured strain and acceleration), and *x* is the state.

In order to apply the subspace method for unsteady loads measurement, the following process is followed.

- Apply known force signal to the CTS at the store CG location
- Record measurements for all sensors
- Perform subspace system identification to determine the state space equations (A,B,C,D) describing the CTS structural dynamics using the force and measurement datasets

C. Measurement Estimation

1. Inverse State Space Method

Once the CTS structural dynamics have been identified, they are used to estimate unknown input forces using measured data. This is currently performed in the frequency domain using the following approach:

$$sx = Ax + Bu \quad . \tag{3}$$

$$(sI - A)x = Bu \quad . \tag{4}$$

$$x = (sI - A)^{-1}Bu \quad . \tag{5}$$

$$y = C(sI - A)^{-1}Bu + Du \quad . \tag{6}$$

$$y = (C(sI - A)^{-1}B + D)u \quad . \tag{7}$$

$$u = (C(sI - A)^{-1}B + D)^{-1}y \quad . \tag{8}$$

where *A,B,C,D* are as determined in system identification, *y* is the measured output, and *u* is the force estimate.

2. Input Estimation Time Domain Method

Development of a linear filter that estimates an unknown input signal to a known dynamic system has been derived for cases with[7] and without direct feedthrough[8]. This filter considers a dynamic system of the form:

$$x_{k+1} = A_k x_k + G_k d_k + w_k \quad . \tag{9}$$

$$y_k = C_k x_k + H_k d_k + v_k \quad . \tag{10}$$

where A,G,C,H define the system dynamics, x is the state, y is the output, d is the input, and w and v are noise processes. The optimal input estimation filter is determined to be:

$$\hat{x}_{k|k-1} = A_{k-1}\hat{x}_{k-1|k-1} + G_{k-1}\hat{d}_{k-1} \quad . \tag{11}$$

$$\hat{d}_k = M_k(y_k - C_k\hat{x}_{k|k-1}) \quad . \tag{12}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_k(y_k - C_k\hat{x}_{k|k-1}) \quad . \tag{13}$$

where the gains (M_k and L_k) are as shown in the reference.

This algorithm has been implemented in Matlab for use on this project. The Verification and Validation section of this report will explore the application of this technique to analytical and experimental data.

3. Inverse Transfer Function Method

An additional method is considered based on the transfer functions determined from the training excitations. The measured acceleration data is converted to the frequency domain. The magnitude at each frequency is multiplied by the inverse of the transfer function magnitude at that frequency.

$$\{Y(\omega)\} = [H(\omega)]\{U(\omega)\} \quad . \tag{14}$$

$$\{U(\omega)\} = [H(\omega)]^{-1}\{Y(\omega)\} \quad . \tag{15}$$

Estimation of unsteady loads based on static calibration reduces equations 14-15 to:

$$\{Y(\omega)\} = [H(\omega = 0)]\{U(\omega)\} \quad . \tag{16}$$

$$\{U(\omega)\} = [H(\omega = 0)]^{-1}\{Y(\omega)\} \quad . \tag{17}$$

III. Demonstration Example

A. Problem Description

A simplified representation of a CTS balance was constructed to demonstrate this method for estimating unsteady loads. In order to develop a cost effective prototype, a simplified version of the CTS balance Phase I demonstration configuration has been developed. The sting was welded into a steel plate in an attempt to create a rigid mounting (Figure 3). The balance was connected to the sting by a threaded connection that can clamp up. A simplified store was connected to the balance by a bolted connection. The store was constructed to allow for adequate structural connections with the accelerometers and load cells.

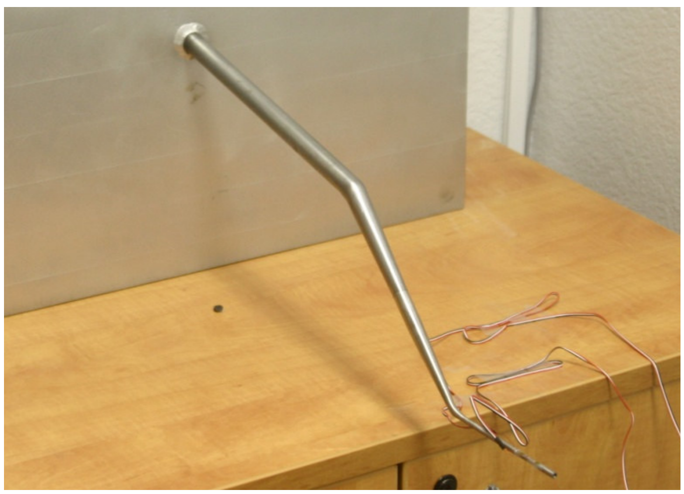


Figure 3: CTS balance configuration for unsteady load estimation demonstration.

B. Finite Element Model

A structural dynamic model has been built to represent the CTS balance configuration. The model is in MSC/Nastran format (Figure 4). The modal results from the structural dynamic model for the CTS balance configuration are shown in Table 1 and Figure 5.

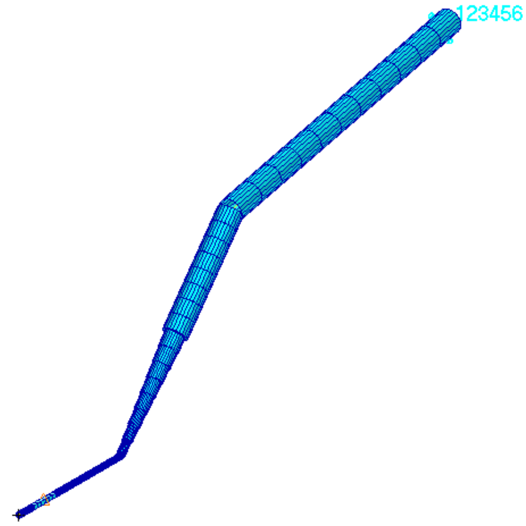


Figure 4: Structural dynamic model of CTS.

Mode Number	Frequency (Hz)	Description
1	44	1st Lateral Bending
2	45	1st Vertical Bending
3	89	2nd Vertical Bending
4	89	2nd Lateral Bending

Table 1: Modal frequencies for CTS.

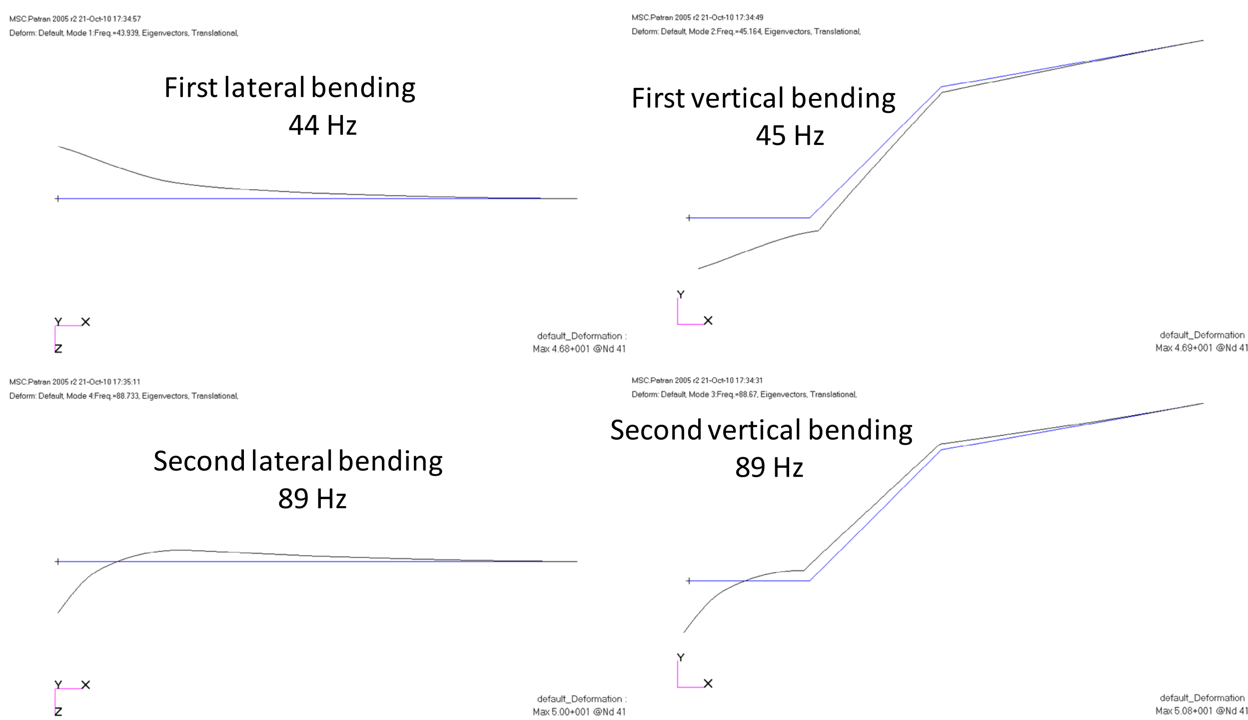


Figure 5: Mode shapes for CTS.

C. Analytical Results

Analytical simulation of the UALM measurement system design process is conducted using the Nastran data. A two input and two output system is considered. The inputs are vertical force and pitching moment at the store CG. The outputs are average and differential vertical acceleration taken at locations one half inch forward and aft of the store CG. Note that the differential acceleration is divided by the length to result in a measurement of the deflection angle in radians. Note that this simulation does not include any unmodeled dynamics, nonlinearities, process noise, or measurement noise. The lack of these factors makes this analytical demonstration less challenging than experimental demonstration.

The equations of motion for the structural dynamic system are written in terms of the generalized deflection (ξ) as:

$$\begin{Bmatrix} \dot{\xi} \\ \ddot{\xi} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{Bmatrix} \xi \\ \dot{\xi} \end{Bmatrix} + \begin{bmatrix} 0 \\ -M^{-1}\Phi_L \end{bmatrix} \{f\} \quad . \tag{1}$$

$$\{\ddot{y}\} = \begin{bmatrix} -\Phi_S^T M^{-1} K & -\Phi_S^T M^{-1} C \end{bmatrix} \begin{Bmatrix} \xi \\ \dot{\xi} \end{Bmatrix} + \begin{bmatrix} \Phi_S^T M^{-1} \Phi_L \end{bmatrix} \{f\} \quad . \tag{2}$$

where M, K, C are the modal mass, stiffness, and damping respectively, Φ_S and Φ_L are the matrices of mode shapes in the sensor and load degrees of freedom respectively, f is the input force, and $\ddot{y}$ is the output acceleration.

The first step is identification of the structural dynamic system. This is performed using a one second training signal with white noise force content for 0.10<t<0.11sec and white noise moment content for 0.20<t<0.21sec (Figure 6a). The simulated acceleration response is shown in Figure 6b. The subspace system identification method described above is used to identify the structural dynamic system. The identified and actual structural dynamic systems are compared in Figure 7. The correlation of the identified and actual systems is excellent.

The next step is to predict the input given acceleration data. Simulation was conducted of a colored noise loading applied for one second. The input estimation time domain method is used to predict the input using only the acceleration data. The measured acceleration data is shown in Figure 8a and the estimated force is shown compared to the actual force in Figure 8b. Note that while the acceleration data clearly shows the presence of structural resonances, the estimated force data shows excellent correlation with the applied force data.

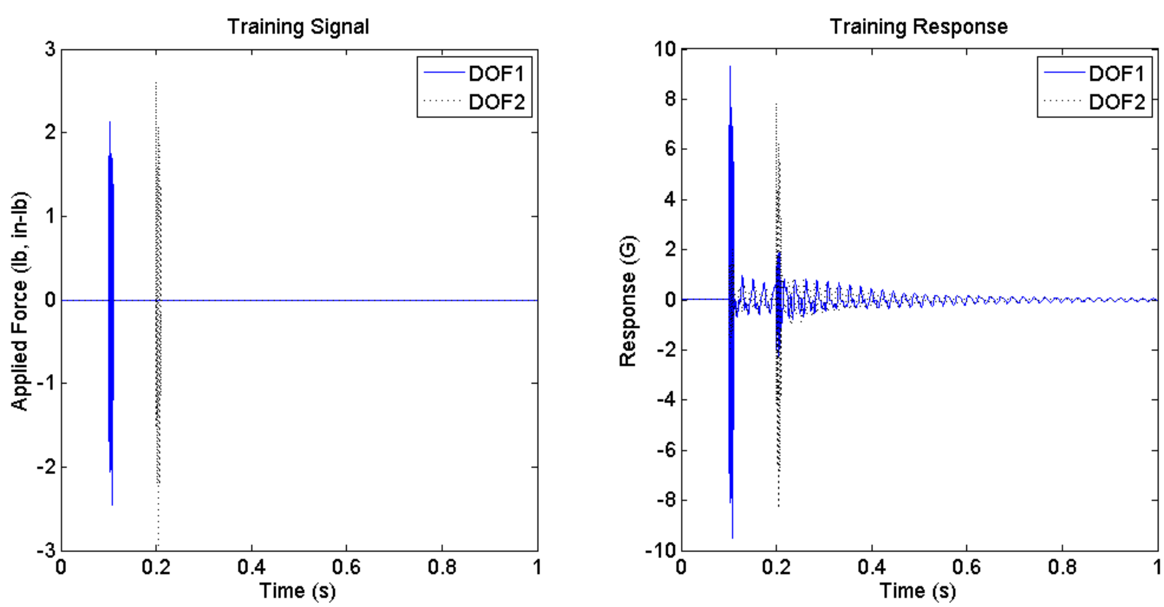


Figure 6: Simulation of training: (a) applied load, (b) response.

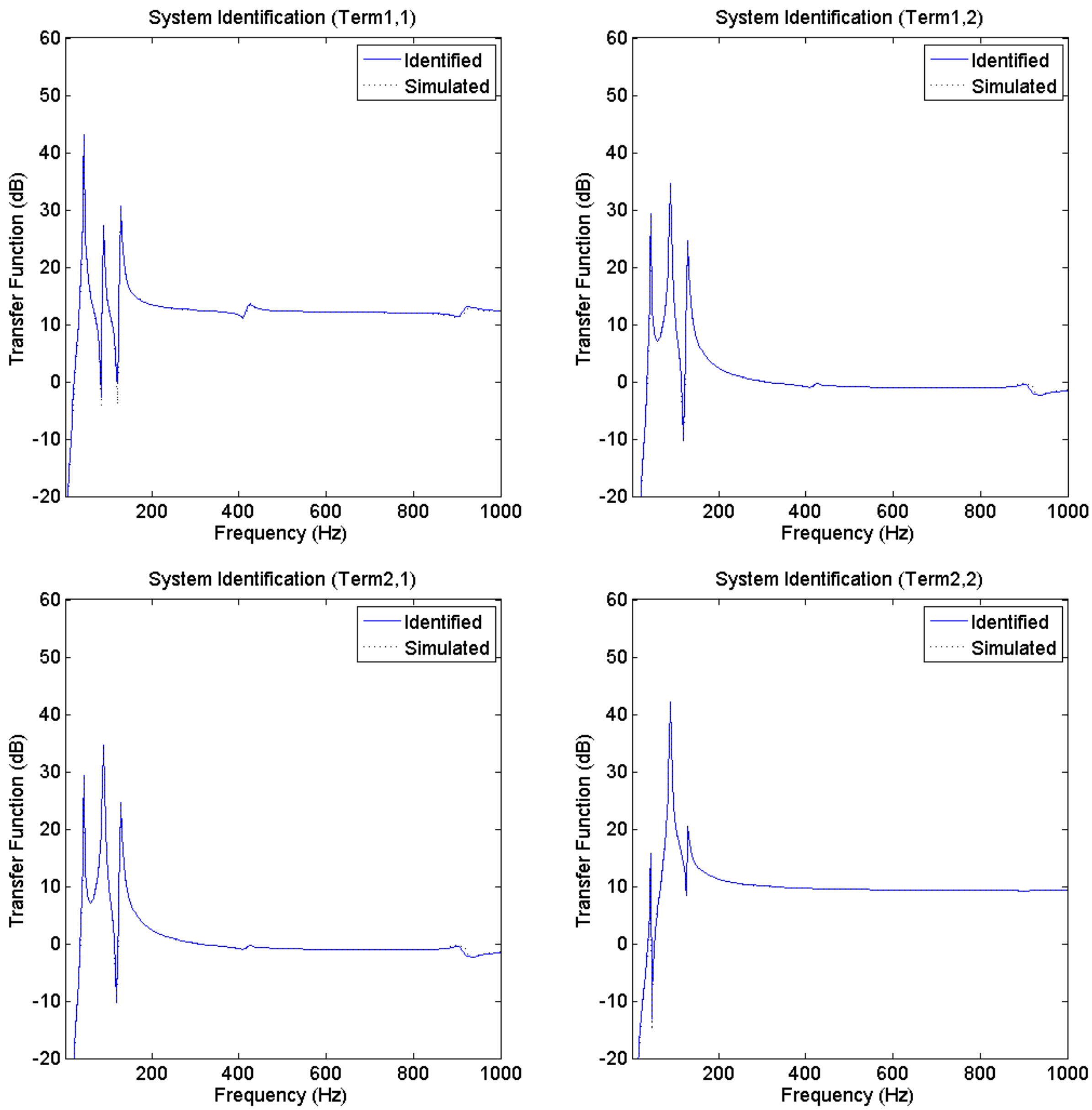
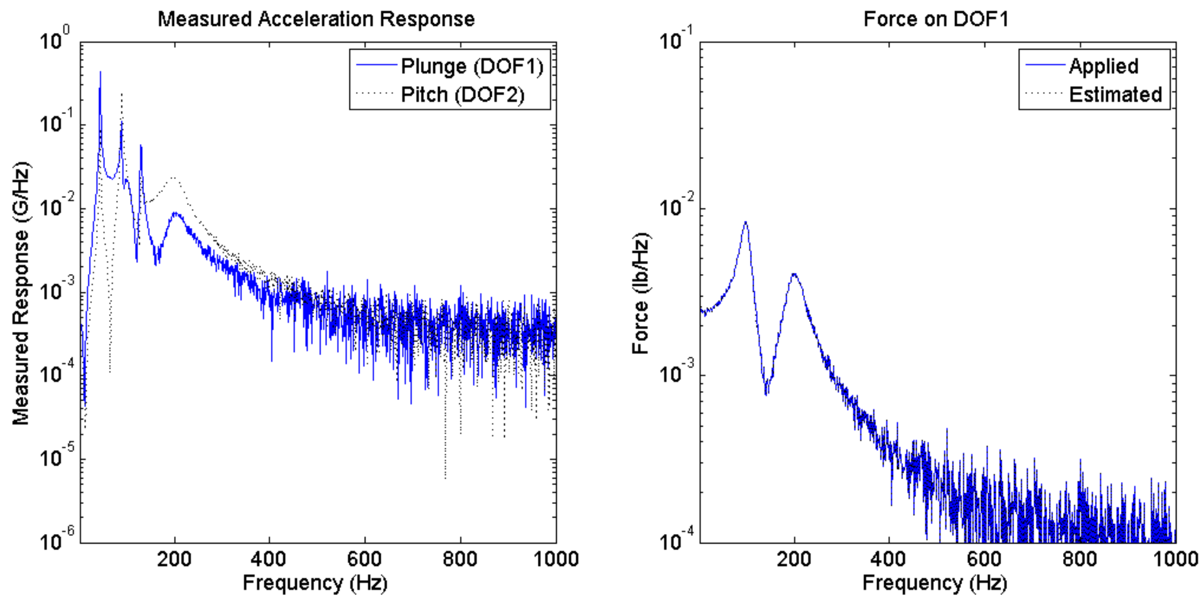


Figure 7: Identified structural dynamic system.



a) b)

Figure 8: Simulation of experiment: (a) measured response, (b) estimated force.

D. Test Setup

In order to conduct the test, the store was instrumented with the following sensors:

- 1 Omega LIVM dynamic force sensor model DLC101-10 with Omega ACC-PSI power supply
- 2 Micro-Measurements strain gages model C2A-13-062LW-350
- 3 PCB Piezotronics accelerometers model 353B18

Excitation was provided with MB Dynamics vibration exciter model PM50A powered by Labworks Inc. amplifier model PA-141. The exciter was connected to the sting with a 19 3/8” rod to reduce moment application at the load cell location. The instrumented store model is shown attached to the vibration exciter in Figure 9.

Data acquisition was performed with National Instruments NICDAQ-9172. The following modules were used:

- NI9233: Accelerometers
- NI9264: Voltage output
- NI9205: Load cell
- NI9236: Strain gages

The sampling frequency was 2000 Hz. 150 seconds of data was collected at each test point.

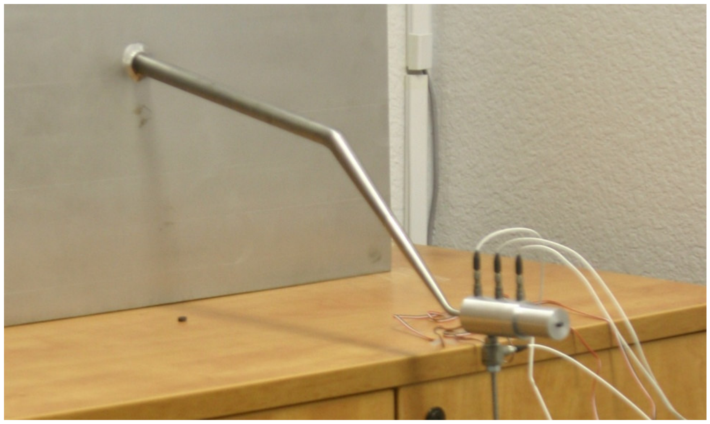


Figure 9: Demonstration including sting, simulated store mass, and load cell.

E. Experimental Results

Input estimation was performed for two store masses with each of the techniques described above. The results are summarized in Table 2. The correlation between the estimated and measured force data is assessed using a "Force Correlation Factor". This factor is calculated by taking the average ratio between the measured and estimated PSD data integrated on ten 100 Hz intervals to determine RMS levels in each frequency band. The ratio is inverted if necessary to ensure that this ratio is greater than 1 for all cases. The closer the correlation factor is to 1, the better the correlation. Note that the best correlation is achieved for acceleration measurement using the inverse transfer function and inverse state space methods. The correlation for these cases is significantly better than the correlation for the static method.

Detailed results for system identification and load estimation of the 0.11 lb store case are shown in the following sections. Transfer functions are plotted to assess the quality of the system identification. Power Spectral Density (PSD) plots are presented to assess the correlation of the estimated and measured force data. System identification results were most successful for cases in which acceleration was measured (Figure 14). Acceleration measurement also enabled improved correlation for force estimation as seen for the inverse transfer function (Figure 15) and inverse state-space (Figure 16) methods.

Store	Measurement	Method	Model Order	Force Corr. Factor	Moment Corr. Factor
0.23 lb	Strain	Static	N/A	22.8	18.3
0.23 lb	Strain	Inverse TF Matrix	N/A	4.6	8.4
0.23 lb	Strain	Inverse State-Space	40	13.2	14.8
0.23 lb	Strain	Time Domain	40	Unstable	Unstable
0.23 lb	Acceleration	Inverse TF Matrix	N/A	2.5	3.1
0.23 lb	Acceleration	Inverse State-Space	40	1.7	2.4
0.23 lb	Acceleration	Time Domain	40	Unstable	Unstable
0.11 lb	Strain	Static	N/A	12.6	11.2
0.11 lb	Strain	Inverse TF Matrix	N/A	4.8	14.0
0.11 lb	Strain	Inverse State-Space	40	14.3	7.0
0.11 lb	Strain	Time Domain	40	Unstable	Unstable
0.11 lb	Acceleration	Inverse TF Matrix	N/A	1.8	5.2
0.11 lb	Acceleration	Inverse State-Space	40	1.9	3.7
0.11 lb	Acceleration	Time Domain	40	Unstable	Unstable

Table 2: Input estimation results.

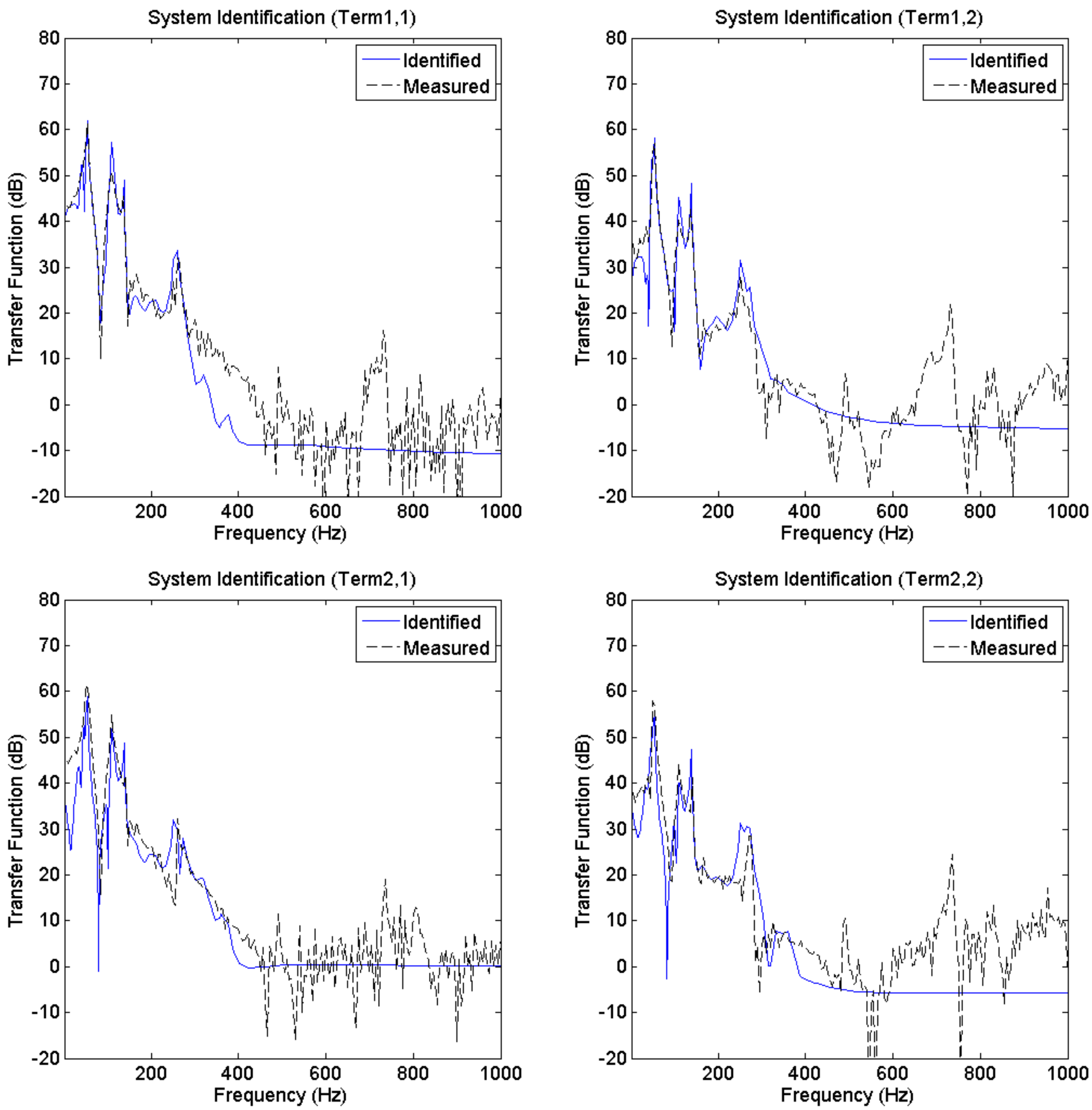


Figure 10: Identified structural dynamic system using strain measurement.

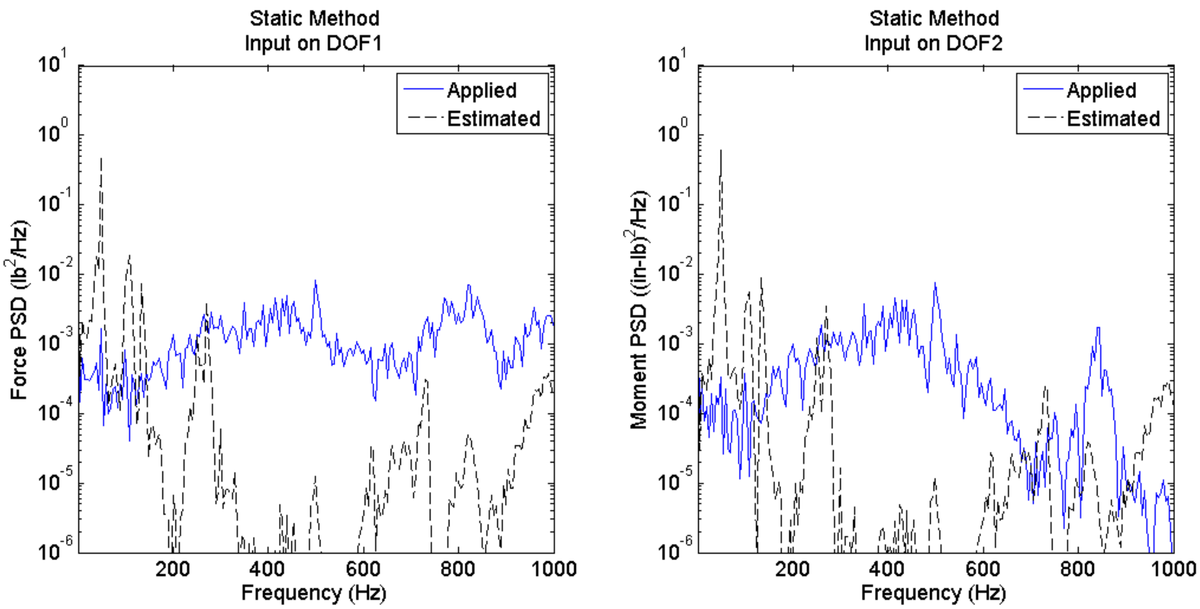


Figure 11: Estimated dynamic loads using the static method with strain measurement.

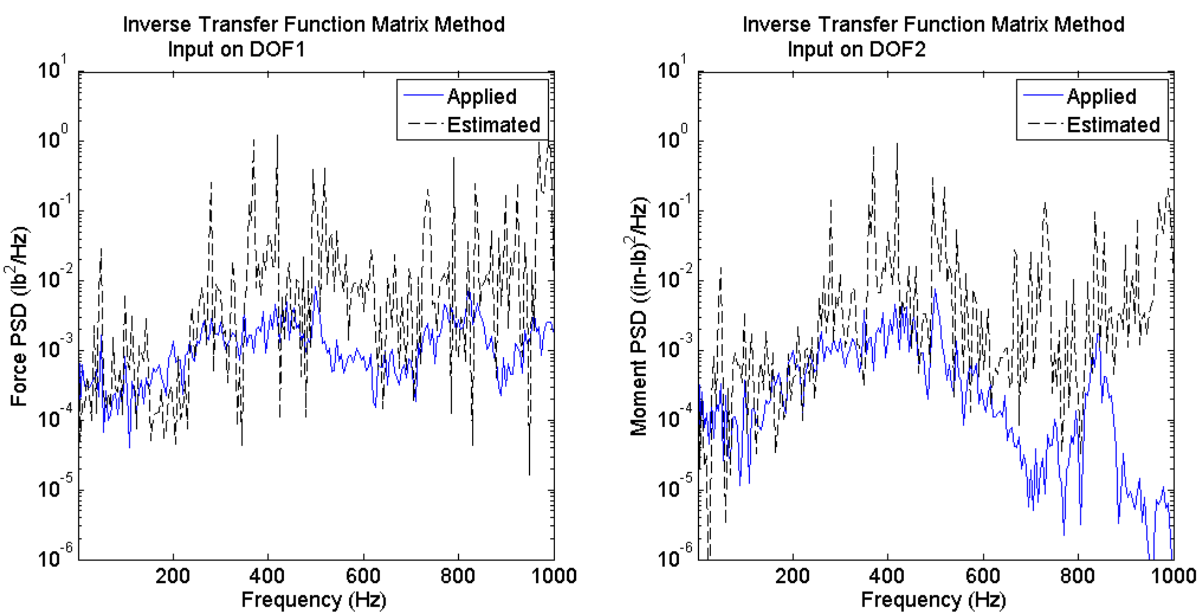


Figure 12: Estimated dynamic loads using the inverse transfer function matrix method with strain measurement.

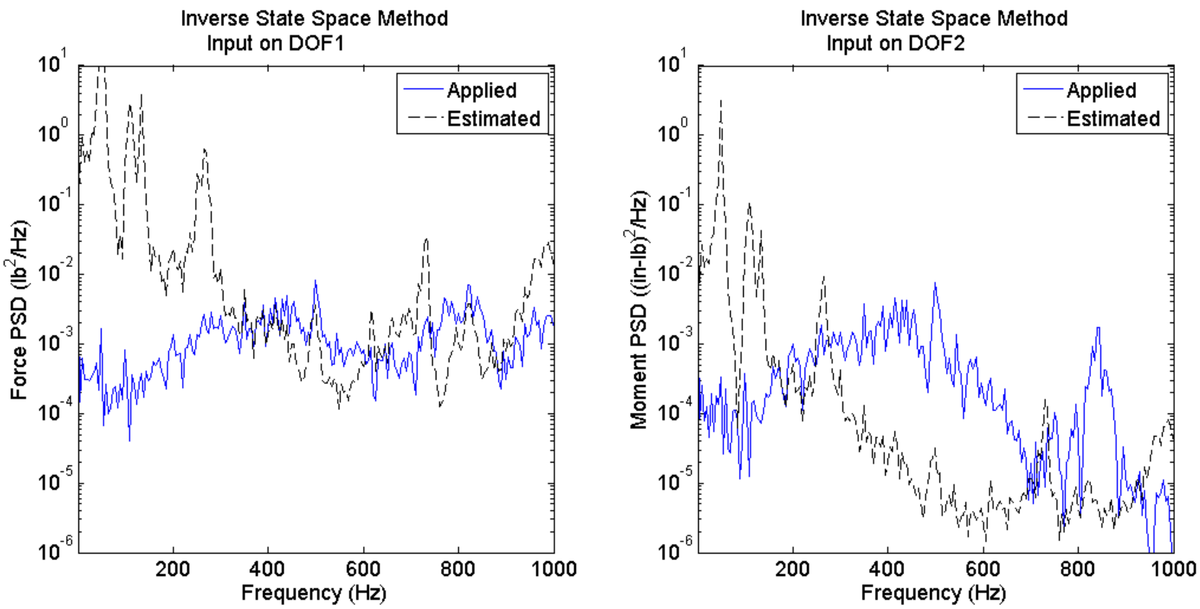


Figure 13: Estimated dynamic loads using the inverse state space method with strain measurement.

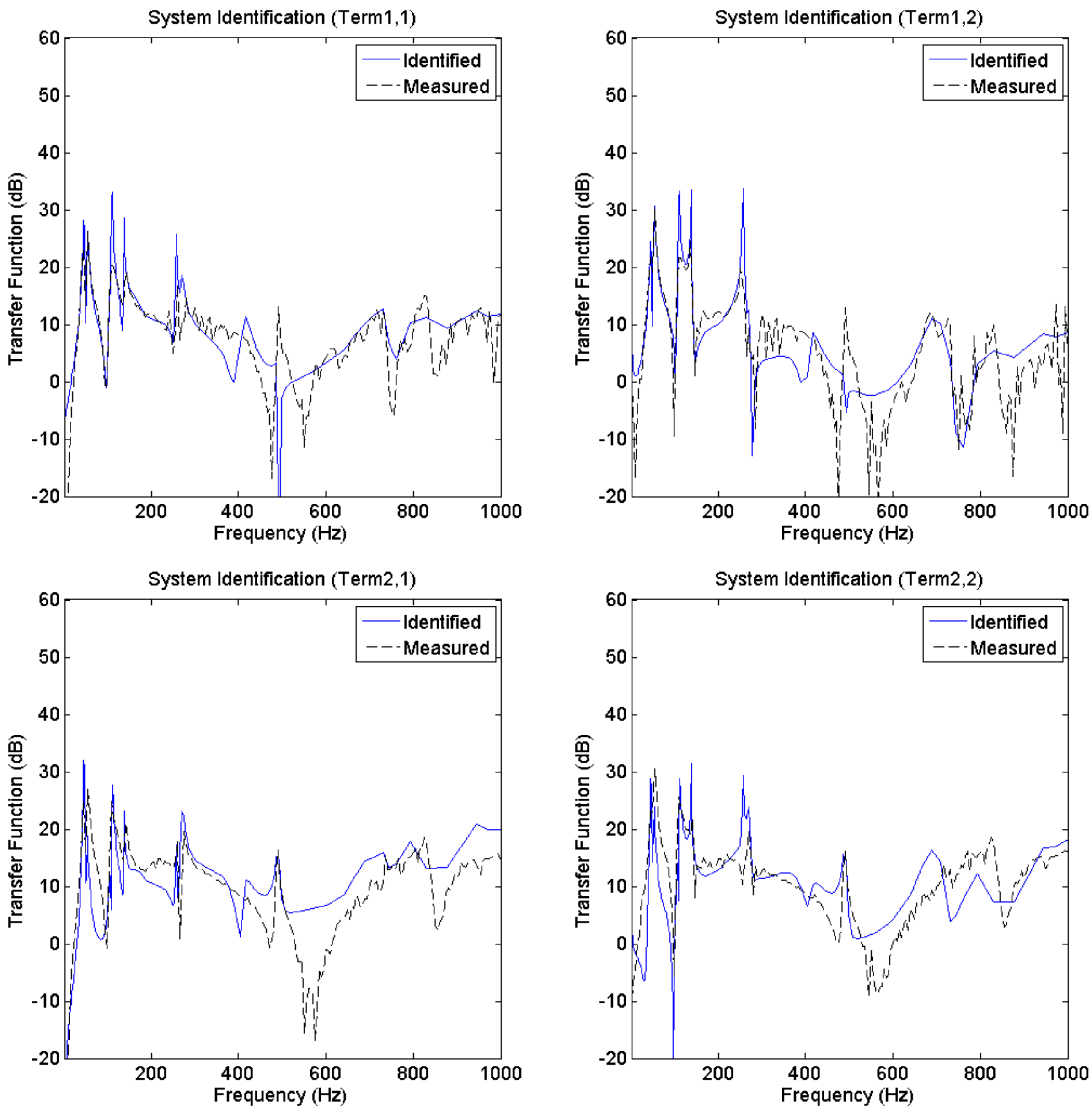


Figure 14: Identified structural dynamic system using acceleration measurement.

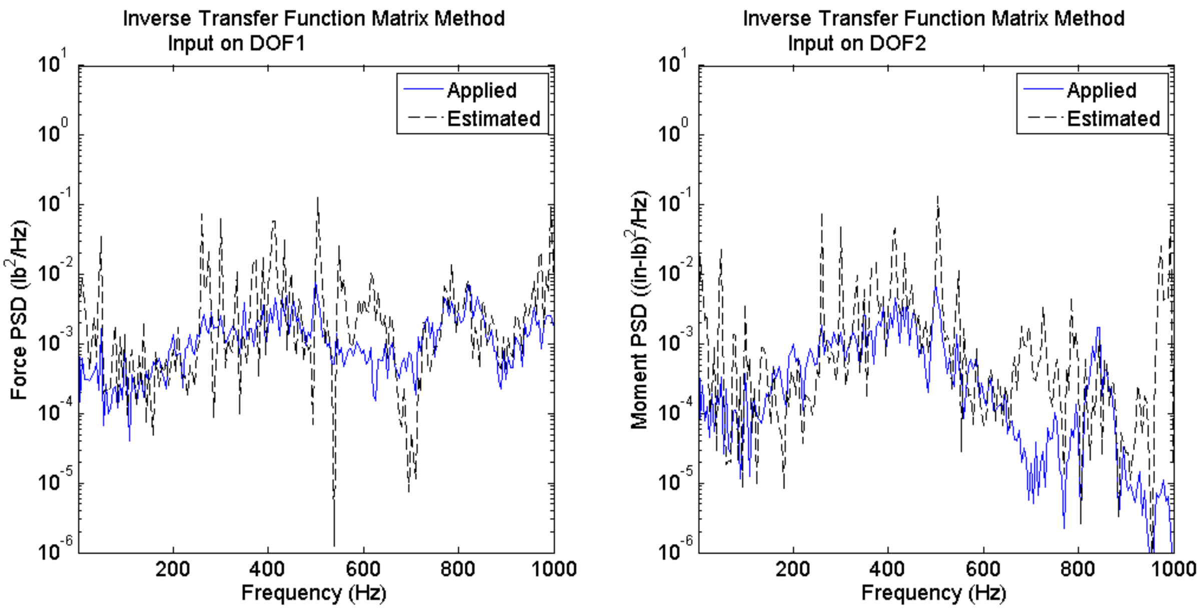


Figure 15: Estimated dynamic loads using the inverse transfer function matrix method with acceleration measurement.

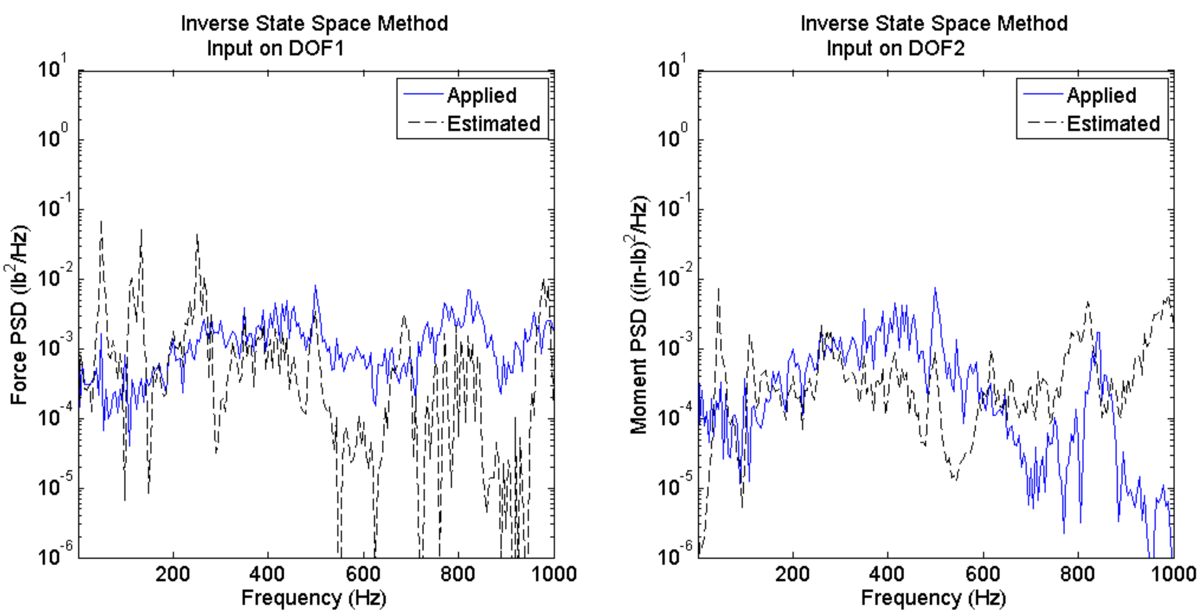


Figure 16: Estimated dynamic loads using the inverse state space method with acceleration measurement.

IV. Conclusion

Unsteady load estimation was demonstrated analytically and experimentally. Analytical demonstration showed nearly perfect correlation between the applied and calculated unsteady loads. For the experimental case, the best correlation was obtained using the inverse transfer function method using accelerometer measurement. All methods showed better prediction of unsteady loads than the approach of using static calibration.

V. Acknowledgement

Funding for this development has been provided by the Air Force at AEDC. The technical monitor is Tyler Neale.

VI. References

1. Roughen, K., Wang, X., Bendiksen, O., and Baker, M. "A System for Simulation of Store Separation Including Unsteady Effects," AIAA-2009-549, 47th AIAA Aerospace Sciences Meeting, Orlando, Florida, Jan. 5-8, 2009.
2. Carman, J. B., Jr., Hill, D. W., Jr., and Christopher, J. P. *Store Separation Testing Techniques at the Arnold Engineering Development Center, Volume II: Description of Captive Trajectory Store Separation Testing in the Aerodynamic Wind Tunnel (4T),* June 1980. AEDC-TR-79-1, DTIC Number AD-A087561
3. Ljung, L. *System Identification, Theory for the User,* Prentice Hall PRT, New Jersey, 1999.
4. Van Overschee, P., and DeMoor, B. *Subspace Identification of Linear Systems: Theory, Implementation, Applications,* Kluwer Academic Publishers, 1996.
5. Peeters, B., Van der Auweraer, H., Guillaume, P., and Leuridan, J. "The PolyMAX Frequency-Domain Method: a New Standard for Modal Parameter Estimation?" Shock and Vibration, Vol. 11, No. 3, 2004.
6. Juang, J.N. and Pappa, R.S. "An Eigensystem Realization Algorithm for Modal Parametr Identification and Model Reduction," Journal of Guidance, Control, and Dynamics, Vol. 8, 1985.
7. Gillijns, S., De Moor, B. "Unbiased minimum-variance input and state estimation for linear discrete-time systems with direct feedthrough," Automatica, Vol. 43, 2007.
8. Gillijns, S., De Moor, B. "Unbiased minimum-variance input and state estimation for linear discrete-time systems," Vol. 43, 2007.