Aeroservoelastic Modeling, Analysis, and Design Techniques for Transport Aircraft

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Abstract

Piloted and batch simulations of the aeroservoelastic response of flight vehicles are essential tools in the development of advanced flight control systems. In these simulations the number of differential equations must be sufficiently large to yield the required accuracy, yet small enough to enable real-time evaluations of the aircraft flying qualities and rapid batch simulations for control law design. The challenge of these conflicting demands is made especially difficult by the limited accuracy of the analytical modeling techniques used, nonlinearities in the quasi-steady equations of motion and by the complex characteristics of the unsteady aerodynamic forces. In this paper, a brief survey of some of the techniques that have been used at Boeing to develop aeroservoelastic math models for control system design and evaluation are presented, along with a discussion of the strengths and weaknesses of the various techniques. The modeling techniques discussed include frequency response fitting methods, rational function approximation methods, and the P-Transform technique. Integration of the aeroservoelastic structural dynamic model with a nonlinear flight simulation is also discussed.

Introduction

Historically, flight control laws have been designed based on the quasi-steady, mean axis flying qualities of an aircraft. However, as airplanes get larger and larger, flexibility and structural dynamics become more and more important. In order to address the influence of aeroservoelastic interactions in large aircraft, it is necessary to include structural dynamic and aeroelastic effects in the simulation tools used for control law design. This results in the generation of a dynamic ASE model with a large number of degrees of freedom for many flight conditions, which creates significant challenges for both the structural and flight controls engineers.

An additional difficulty in aeroservoelastic analysis stems from the fact that the modeling and analysis techniques that are most applicable to aeroelastic loads or flutter analysis are not necessarily those that are most useful for control law design. Traditionally, aeroelasticians have modeled the flexible aircraft in the frequency domain using modal degrees of freedom and generalized mass, generalized stiffness, and frequency dependent generalized aerodynamic matrices. On the other hand, modern control theory is based primarily on the state-space approach, in which the aeroelastic airplane must be modeled as a first-order system of linear ordinary differential equations in the time domain. In addition, the aeroelastician typically works in a mean flight path coordinate system, and the flight controls engineer in a body axis coordinate system.

The final challenge is to ensure that throughout the transformations from frequency domain to time domain, and from one axis system to another, the models remain consistent. This ensures that when a control law is designed based on the time domain state-space model, the same control law can be input into the aeroelastician's frequency-domain analysis and comparable results can be expected.

Modeling of Structural Dynamics & Unsteady Aerodynamics

Three main classes of time domain mathematical modeling techniques are discussed, including the P-transform [1,2,3], frequency response fitting techniques [4], and use of rational function approximations [5,6,7,8,9,10,11,12]. In addition, the application of promising aerodynamic model reduction techniques to the aeroservoelastic model reduction problem are discussed [13,14,15,16,17,18]. Integration of these time domain aeroelastic modeling techniques with nonlinear rigid body and static aeroelastic equations of motion to develop a universal aeroelastic simulation model for use by both aeroelasticians and flight control designers will also be addressed.

P-Transform Technique

Heimbaugh [1] developed a formulation of the aircraft equations of motion (EOM) that provides an accurate modeling of the flexible aircraft (including the unsteady aerodynamic forces) without aerodynamic lag terms. The formulation is analogous to the process that is employed in structural dynamics to reduce the number of degrees of freedom (DOF) and transform the equations into modal coordinates using the Galerkin approach. Essentially, it can be expressed as:

1. Write the frequency-domain equations of motion of the aircraft:

   $$\left[Ms^2 + \left(B - \tfrac{\rho c V}{4} A^{i/k}\right) + \left(K - \tfrac{\rho V^2}{2} A^r\right)\right] x(s) = F(s)$$

   where M, B, and K are the mass, damping, and stiffness matrices, A is the generalized Aerodynamic

Influence Coefficient (AIC) matrix (real and imaginary parts), x is the generalized deflection, F is the generalized force, and s is the Laplace variable.

2. Compute the "important" eigenvalues & eigenvectors using a standard flutter solution technique. Here it is assumed that the eigenvalues found in the flutter solution are those important in the response.
3. Construct a time domain state-space model (A, B, C, D Matrices) based on the known eigenvalues & eigenvectors.

The assembly of the state matrices and associated input/output matrices from the aeroelastic eigensolution is roughly equivalent to the generation of a "quasi-unsteady" aerodynamic fit for each aeroelastic mode. For low damped modes (sharp modal peaks) this is a very good approximation. For lightly damped modes (more gradual, wider peaks) the quasi-unsteady aerodynamic fit is only exact at the peak of the mode, thus the accuracy is reduced. This technique has been shown to be extremely accurate when the input excitation is a control surface deflection. For gust inputs, the P-Transform was augmented with aerodynamic lag states based on a Rational Function Approximation (RFA) originally developed by Sevart [8] and Roger [5]. The p-transform technique was used to generate aeroelastic models for production aircraft programs and advanced design studies at Boeing in Long Beach for many years. Some of the applications of this technique include an advanced design of a DC-10 stretch aircraft, the C-17 program, and the MD-11 program.

These models were generated using beam structural models, unsteady aerodynamics from the Doublet Lattice Method (DLM) [19,20], and small perturbation assumptions. Typically, it is difficult to accurately model many of the rigid body aeroelastic modes of the aircraft using these types of modeling techniques. In particular, neglecting drag and other "second order" aerodynamic effects can significantly effect several of the modes (Dutch Roll and Phugoid for example). An example of the sensitivity of the Dutch Roll frequency and damping to a 10% change in global aircraft weight, inertia, and aerodynamic forces is shown in Figure 1. The nomenclature in this Figure is consistent with that used in [21]. Heimbaugh accounted for these effects in a simplified manner by adding modal stiffness and damping terms to the equations of motion to account for the neglected terms. These terms were locally linearized based upon the trimmed aircraft attitude and had to be recomputed for each flight condition.

In summary, there were many advantages to this type of approach. These advantages included:

- Elastic aircraft formulation is consistent with that used in the flutter and dynamic loads analyses. Roots from Aeroservoelastic models will be consistent with those generated from analytical flutter models.
- The technique could accurately capture the correct mode shapes, frequency, and damping values of both the rigid body and flexible modes.
- The technique provided a high level of accuracy for low-damped modes.
- Additive increments used to accurately represent the rigid body motion of the aircraft also were used directly in the generation of physical DOF responses since they are generated from the modal responses.
- Models could be reduced further while retaining the accuracy through the Guyan reduction.

Disadvantages of this approach included the following:

- Due to the nature of the RFA used in this technique for the gust forces, the gust fit was only accurate for lower frequency gust excitations.
- The technique was very cumbersome to use because there were many convergence problems associated with the p-k type of flutter solution.
- Due to the linear nature of the analysis, there was difficulty in some cases to generate locally linear additive corrections that were appropriate. There are many terms that significantly influence the rigid body motion of the aircraft that can only realistically be accommodated in a nonlinear manner.
- Supplemental corrections that were applied in an additive manner to the generalized equations of motion were never distributed and thus were not reflected directly in the distributed loads.
- An unknown amount of error existed for aeroelastic modes with a high level of damping.

Typical results from this analysis technique are included in Table 1 and Figure 2. Table 1 compares phugoid and short period frequency and damping values for a large commercial transport aircraft. This table lists both the result that was obtained using the traditional stability and control analysis equations (Target) and that obtained using the additive correction terms in the structural equations of motion (P-Transform). This comparison illustrates that the accuracy of the augmented equations of motion were within 1-2% in frequency and 2-4% in damping.

Figure 2 shows the magnitude of the inboard wing bending moment due to a vertical gust for the same commercial aircraft at another flight condition. The graph compares frequency responses from two analyses, one performed using a traditional transcendental frequency domain analysis approach (solid line) and another using the P-Transform technique (dashed line). It is observed that the results are fairly close for the short period mode (~0.3 Hz.) and for the first and third elastic mode (~1.3 and ~3.0 Hz.). Thedata for the second flexible mode are not as accurate asfor the first and third. As indicated above, this is due to the representation of the gust forces using the Sevart/Rogers RFA.

FAMUSS

Pitt and Goodman [4] developed the Flexible Aircraft Modeling Using State Space (FAMUSS) technique at Boeing in St. Louis in the late 80's and early 90's. This tool was used in Long Beach for development of maneuver and gust load alleviation, vibration control, and flutter suppression systems in several advanced design studies on both commercial and military aircraft projects.

This tool required input of frequency response data from an outside source (i.e. a transcendental frequency-domain analysis) and used a linear least squares fit to generate a rational polynomial representing the frequency response function. These polynomials were then converted to state space (in block-diagonal form) using simple algebraic techniques. An option was available that allowed the poles of the system to be pre-defined and constrained. A nonlinear least squares approach was also available to improve the polynomial representation as well as an option for frequency dependent weighting to improve the fit in a specific frequency band.

At that time, the preferred process used by Boeing in Long Beach was to generate frequency response data for each input/output pair included in the final state-space model. These frequency responses were computed using the traditional frequency-domain tools that were used for flutter or dynamic loads. Aeroelastic roots (frequency and damping) were calculated using a traditional p-k type of flutter solution. These frequency responses and aeroelastic roots were then input into FAMUSS. The aeroelastic roots were used to generate the denominator terms in the polynomials in a manner analogous to pole-placement techniques. The numerator terms (which result in the input and output matrices) were then generated using the linear least squares approach.

Accuracy of this technique for computing loads responses to gust inputs was limited. Aircraft physical responses due to gusts are computed through the superposition of the modal responses. The aircraft loads, however are a result of both external forces and forces developed due to aircraft rigid body or flexible motion. It was speculated at that time that the inaccuracies were primarily due to a poor representation of the extrenal force. It was further speculated that this inaccuracy was caused either by delays introduced into the equations of motion from the gradual penetration of the aircraft into the stationary gust field, or by inaccuracies at frequencies where the external forces were prevalent. Through a trial and error fashion, it was finally concluded that the introduction of an additional three aerodynamic roots that were not constrained or preselected significantly increased the accuracy of the state-space model with gust inputs. The location of the additional roots was determined through a linear least squares technique.

In summary, there were many advantages to this type of approach. These advantages included:

- The procedure was simple and very robust. The code was very user friendly and included many graphical techniques to review the accuracy and restart capabilities to further improve the model.
- Due to an improved formulation of the iterative P-K solver [22], we were able to obtain solutions for conditions in which the early P-Transform technique has failed to converge.
- Frequency responses could be computed for the entire system and the user could select the model size inside of FAMUSS based upon the needs of the control analysis being performed.

Disadvantages of this approach included the following:

- The accuracy of the rigid body modes in the procedure as illustrated above was subject to the accuracy of the RB mode in the analysis that produced the transfer function. The technique had not been integrated with a more accurate technique for defining or modifying the rigid body response.
- The accuracy of the aeroelastic model for systems where there were a large number (>5-10) of inputs or outputs was significantly reduced. As the number of inputs/outputs is increased the accuracy is decreased unless the number of states (roots) is increased. Ultimately, for many of the practical problems experienced, the number of roots required to obtain an acceptable level of accuracy was very large.

Example frequency responses from this analysis technique are included in Figures 3-4. These responses resulted from a study of gust load alleviation on a large commercial transport aircraft. Due to the large number of control studies performed at that time on this aircraft, a relatively small aeroelastic model was requested to allow for a number of control configurations to be rapidly evaluated. Since the primary interest in this analysis was the low frequency response (including the first couple of wing elastic modes), the number of modes retained in the analysis was kept to a minimum (~6 modes total).

Figure 3 illustrates a comparison of the aircraft CG pitching response due to elevator deflection. In this Figure, the response from the traditional frequency domain analysis is represented by the solid line. The dashed line illustrates the frequency response resulting from the FAMUSS aeroelastic model. As illustrated in this Figure, the short period mode and the first two flexible modes are accurately represented. Accuracy was limited for some of the higher aeroelastic modes in this model.

Figure 4 shows the CG Pitching response due to a vertical gust. Once again, the solid line represents the traditional frequency domain analysis and the dashed line illustrates FAMUSS model. For this response the model was much more accurate and fairly accurately represented the modal characteristics for all of the aeroelastic modes in the model.

Rational Function Approximation (RFA) Techniques

Over the past 25 years, many researchers have investigated the use of RFA techniques to represent unsteady aerodynamic forces in aeroelastic analytical models. The aircraft equations of motion have generally been formulated using a modal approach to represent the structural dynamics. The unsteady aerodynamic forces are also generated in modal coordinates and are represented by a rational function in frequency. Since the aerodynamics are represented by a rational function, they can be analytically transformed into the time domain using Laplace transform techniques. A final set of time domain equations can then be formulated and cast in state-space form.

Some of the initial studies included Sevart [8], Roger [5], Edwards [9], and Vepa [10]. These approaches differ in the form of rational function that is used to represent the aerodynamic forces. All of these techniques added a large number of aerodynamic states to the aeroelastic equations of motion. Generally researchers have predefined the poles of the rational function and the numerator coefficients have been determined using linear least squares techniques. Researchers have also investigated optimization of the pole location and other techniques to improve the aerodynamic representation and reduce the number of equations that are required to achieve the required level of accuracy.

The state-space form of the equations of motion using the RFA techniques listed above result in large partitions of null value coefficients. To reduce the size of the aeroelastic system, Karpel [11] developed a form of RFA that reduces the sparse nature of the matrices and the number of states accordingly. This method used convergent iteration techniques to optimize the aeroelastic models given a reduced number of states. This method was named the "Minimum State Method".

Boeing Long Beach has investigated the use of RFAs in the generation of aeroelastic modeling for many years and has kept abreast of changes in this area. Although there was great desire to reduce the size of these aeroelastic models, robustness issues associated with the convergence of accurate minimum state models were never quite resolved. Boeing research included the usage of RFA approximations for both motion-induced unsteady aerodynamic forces as well as the gust induced external forces. Research results concluded that in order to obtain the robustness required in a production engineering environment, the usage of a method such as that developed by Roger [5] was required. The research also concluded that in order to accurately model gust forces on large transport aircraft, an RFA that explicitly captured the time lag associated with the penetration of the aircraft into the gust field, was required. Some of the latest elements of this work are published in technical reports and papers from Dykman [6] and Goggin [7].

In summary, there were many advantages to this type of approach. These advantages included:

- The accuracy of the resulting aeroelastic model is very high.
- The robustness associated with the use of some of the RFA techniques is very good. Aeroelastic models can be generated without a significant amount of user intervention.

Disadvantages of this approach included the following:

- The resulting aeroelastic model is very large and could prohibit quick studies and real-time simulation.
- The accuracy of the rigid body modes was subject to the accuracy of the aerodynamics used in generation of the RFA. The technique had not been integrated with a more accurate technique for defining or modifying the rigid body response.

As stated above, accuracy of these RFA techniques for state space models is very good. For the examples shown here, a Rogers RFA with 4 aerodynamic lag states per mode was used to represent the motion dependent aerodynamic forces. A Dykman [6] gust fit was used with a total of 12 gust aerodynamic states (for the complete model, not per mode) to represent the external gust forces. In the gust RFA, the 12 states are comprised from an RFA with three explicit time delays and four repeated roots per delay. An example of the external gust force representation is included here in Figure 5.

Figures 6 and 7 also illustrate the accuracy that can be obtained from a model of this type. Frequency responses comparing the traditional transcendental frequency response technique (dots) is compared to that using the RFA state space model (solid line). Figure 6 illustrates a bending moment response in the wing due to a gust input. Figure 7 illustrates a horizontal tail root shear due to the same gust input. As shown in these Figures the correlation is excellent.

Figure 8 illustrates a time history of the horizontal tail root shear due to a very short 1-cosine vertical gust. This is a case where the gust force representation developed by Dykman was critical. This was a challenge due to the aft location of this component load on this large transport, and the high frequency excited by these types of gust patterns that are specified in the commercial aircraft criteria.

Other Reduced Order Modeling Techniques

All of the techniques described above can essentially be viewed as model reduction techniques where a dynamic system with a high order (due to the transcendental frequency-domain aerodynamics) is approximated by an "equivalent" low order system. Since the unsteady aerodynamic models used by aeroelasticians (and therefore, by aeroservo-elasticians) have traditionally been frequency domain methods based on the linear potential equations (such as the doublet-lattice method), the model reduction techniques that have received most of the attention in the past are the frequency domain methods described above. However, the exponential growth of computer capability (coupled with the exponential decay of computer cost) is paving the way for using nonlinear unsteady aerodynamic tools based on the finite difference (or finite volume/finite element) method in aeroelasticity. This has the potential for improving the accuracy of the dynamic aeroelastic models used in aeroservoelasticity, but introduces some significant problems in formulating reduced order models suitable for control law design and real time simulation.

Recently, several researchers have started to consider the problem of forming reduced order unsteady aerodynamic models based on unsteady CFD models. Three approaches that have received considerable attention recently are (1) eigenvalue based methods, (2) balanced reduction methods, and (3) system identification methods.

In the eigenvalue based model reduction methods, the eigenvalues and eigenvectors of the unsteady flowfield are computed and used as a basis for model reduction [15]. Those eigenmodes that are "important" (usually quantified by the low frequency or lightly damped eigenvalues) are then retained while the "unimportant" modes are truncated or residualized. This approach is almost identical to the modal truncation approach to model reduction that has been applied for many years in structural dynamics, and has shown great promise for model reduction of unsteady aerodynamic systems.

The balanced reduction methods, on the other hand, are based on the concepts of controllability and observability of the unsteady aerodynamic model. In these techniques, the aerodynamic states that are highly controllable (i.e. those that are easily excited by airplane control surface, rigid body, or structural deflections) and at the same time highly observable (i.e. those that, once excited, induce significant loads on the structure) are retained. The balanced reduction approach is less physically intuitive than the eigenvalue based techniques, but has the potential for producing smaller and more accurate reduced order models [13,14].

Both the eigenvalue based methods and the balanced reduction methods suffer from a serious drawback in that they require extensive modifications to the CFD code in order to generate reduced order models. This difficulty is avoided if a system identification approach is used in which the CFD code develops time histories of the unsteady aerodynamics, and an external code is used to process the time history data and generate a reduced order model of the unsteady aerodynamics. The K-L method [16,17] is one such technique that develops an eigenvalue-based reduced order model using a system identification approach. Another approach under development are the impulse response based techniques [18], which directly identify impulse responses of the unsteady aerodynamics. The impulse response approach has not been shown to develop models of low enough order to be useful for control law design. However, this approach has been shown to capture some of the aerodynamic nonlinearities inherent in transonic flow, which could substantially increase the range of applicability of the reduced order models.

Boeing has investigated several of the advanced model reduction techniques described in this section, and has applied them to several configurations with good success. However, none of these techniques is mature enough for use in a production environment. Each of the techniques described has its own advantages and disadvantages, and it is not yet clear which approach is best. It is safe to say, however, that they will probably change the face of aeroservoelasticity as they mature, and as the affordability of computational power increases.

Recent Improvements

New transport aircraft designs like the High Speed Civil Transport (HSCT) and the Blended Wing Body (BWB) are more challenging from an aeroservoelastic perspective than conventional configurations. In the case of the HSCT, there is not only the possibility of gaining benefits from load alleviation systems for minimizing gust and maneuver loads, but significant benefits could also be realized through using flutter suppression. In addition, significant ride quality/flying qualities issues arise due to the long, slender fuselage with its associated low bending frequencies. For the new class of BWB configurations, pitch control is obtained by deflecting trailing edge control surfaces (in contrast to a conventional transport configuration, where pitch control comes from a horizontal tail). It is therefore important to include the interaction between the pitch command and the wing bending modes in control law design.

In order to address these new aeroservoelastic challenges and to improve accuracy on conventional configurations, some recent improvements have been made to the ASE modeling techniques described above. The recent modeling improvements have focused on the P-Transform method for two main reasons: (1) acceptable accuracy can be obtained with very low order P-Transform ASE models, and (2) the poles of the P-Transform model are consistent with the P-K flutter results (which is especially important in configurations with lightly damped modes such as the HSCT).

The enhancements that have been made to the P-Transform method in recent years reduce or eliminate many of the disadvantages highlighted above. The most significant improvements are:

- Integration with Nonlinear Simulation. A large portion of the labor required to generate P-Transform ASE models was associated with generating the additive corrections necessary to accurately model the rigid body modes. This problem has been solved by separating the equations of motion into two parts; one describing the linear quasi-steady response and the other containing the dynamic increment. The quasi-steady equations are then discarded in favor of the more accurate nonlinear 6 DOF simulation typically used in flight controls. The resulting ASE models include the best possible (fully nonlinear) model of the rigid body modes, while including linearized structural dynamics and unsteady aerodynamics through the P-Transform technique.
- Integration with improved P-K Solvers. The convergence problems that caused difficulty in using the P-Transform technique were due to the state of the art in P-K flutter solvers at that time. A tight integration of the P-Transform process with the P-K flutter solver in MSC/NASTRAN [22] has significantly improved this situation.
- Modification of the P-Transform technique to compute structural loads (i.e. wing bending moments or hinge moments) in a manner consistent with the quasi-steady nonlinear simulation.
- Improved modeling of gust aerodynamic forces. An improved RFA technique using explicit time lags [6] for the gust aerodynamics has been implemented, significantly improving the accuracy of responses due to gust excitation.

Several tests were performed to verify that the improved P-transform technique was implemented correctly in the simulation. One such test compared the roots of the linear model with roots obtained from the simulation when it was linearized about the trim point. In addition, various mean-axis response variables were computed at the static trim condition to demonstrate that they were unaffected by the superimposed dynamic increments. Figure 9, showing the roots associated with longitudinal motion, confirms that eigenvalues of the flexible modes in the linear analysis are close to the ones obtained for the simulation model at a banked turn as well as at a level flight condition. The nonlinearity of the 6-DOF simulation can be observed in the changes in the rigid body eigenvalues for different trim conditions. A similar correlation was obtained for roots associated with lateral motion.

The improved P-transform was validated in several other ways. Time response comparisons were made with a version of the RFA technique developed in [12]. Representative results are shown in Figure 10 for an advanced transport aircraft at a Mach number of 0.65. The illustrated response was computed for a horizontal tail doublet input of ± 2 degrees amplitude and a period of 5.0 seconds starting at 2.5 sec. We note that correlation between the two methods is excellent.

The improved method was also evaluated through comparison with MSC/NNASTRAN frequency domain solutions. Figure 11 shows magnitude and phase of the acceleration responses for an advanced transport aircraft flying at sea level and at a Mach number of 0.4. We observe that the two solution techniques yield practically identical magnitude results up to a frequency of 7.0 Hz. Discrepancies above that frequency are explained by differences in the modeling of the elevator surface. A rigid control surface mode is used to generate the P-Transform input whereas a more realistic, flexible elevator model provides the excitation force in the MSC/NASTRAN analysis.

Conclusions

Experiences derived from several transport aircraft programs at Boeing led to a continuous search for, and development of, accurate techniques for ASE modeling and simulation. All of the methods discussed here have several advantages as well as disadvantages. We found that some of the disadvantages of the early P-Transform technique could be removed by separation of the EOMs into two parts, one describing the quasi-steady motion and the other involving the structural dynamics of the aircraft. This development allowed the model to be linked to the nonlinear 6DOF simulation used for analysis and design of advanced flight control systems.

The refined P-transform technique is based on a unique formulation that preserves the roots of the dynamic aeroelastic system and eliminates the need for auxiliary state variables to describe the unsteady aerodynamics. It has provisions for control surface as well as atmospheric gust inputs. Comparisons with other solution techniques were used to validate the method. Our analytical results demonstrate excellent correlation with structural response data (accelerations, rates and displacements) obtained from the transcendental frequency-domain solution. Further work is required to evaluate the accuracy of external loads generated by turbulence.

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Root	Natural Frequency (Hz)		Damping (% Critical)	
	Target	P-Transform	Target	P-Transform
Phugoid	0.0115	0.0114	39.13%	40.41%
Short Period	0.2585	0.2638	31.69%	32.72%

Table 1: Accuracy of P-Transform in Matching Rigid Body Modes of a Transport Aircraft.

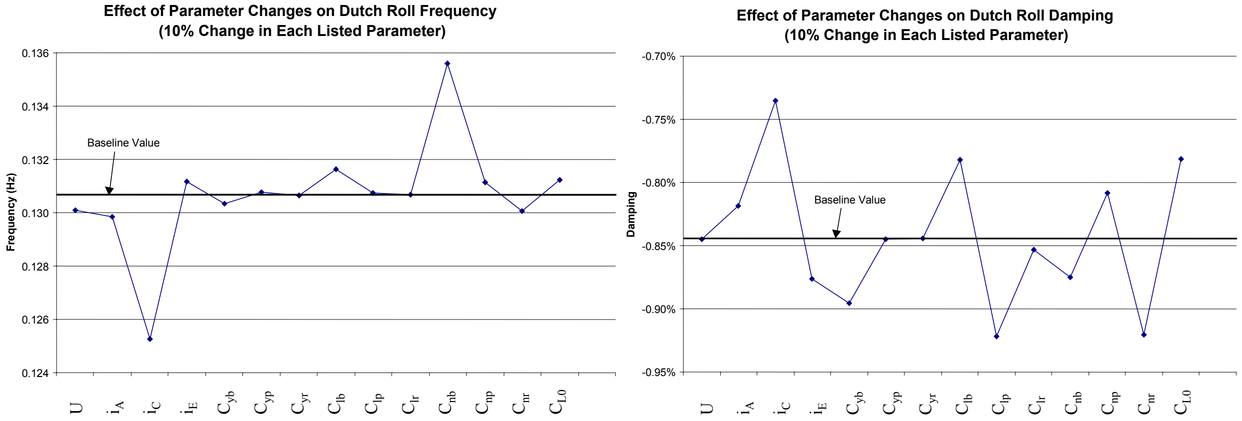

Figure 1: Variation in Dutch Roll Rigid Body Mode Frequency and Damping With 10% Changes in Various Parameters.

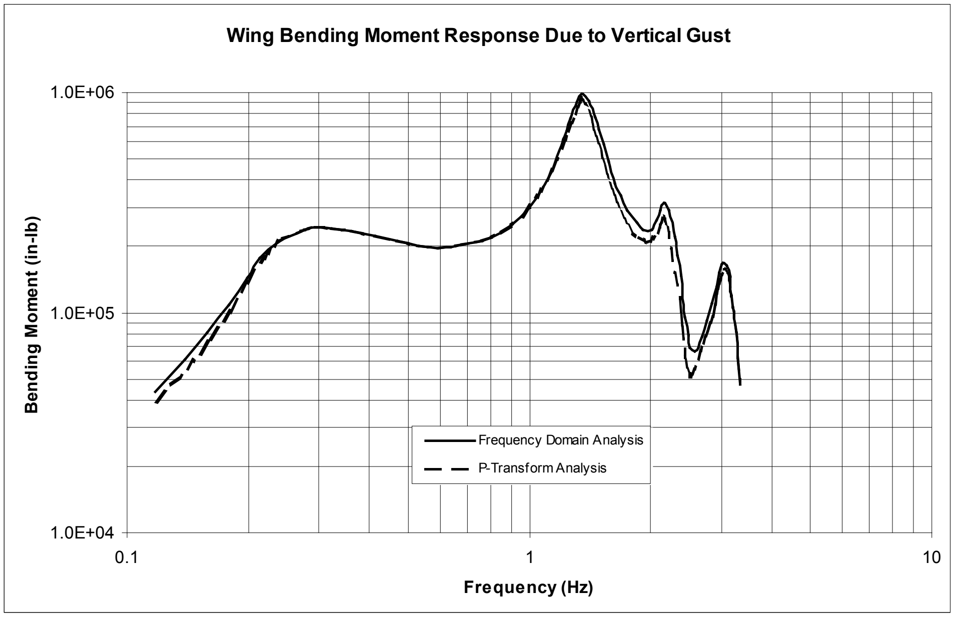

Figure 2: Accuracy of the P-Transform Method for Computing Wing Bending Moments Due to Vertical Gust.

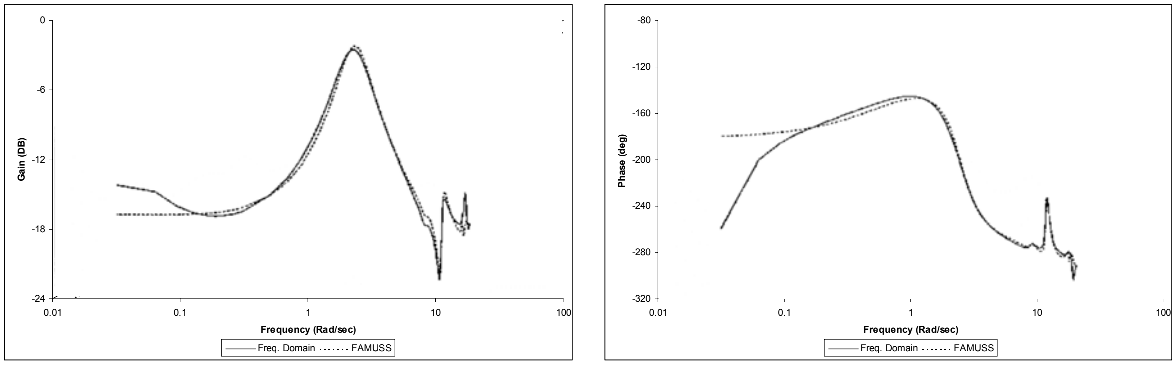

Figure 3: Application of the FAMUSS process to compute CG Acceleration Due to Elevator Excitation.

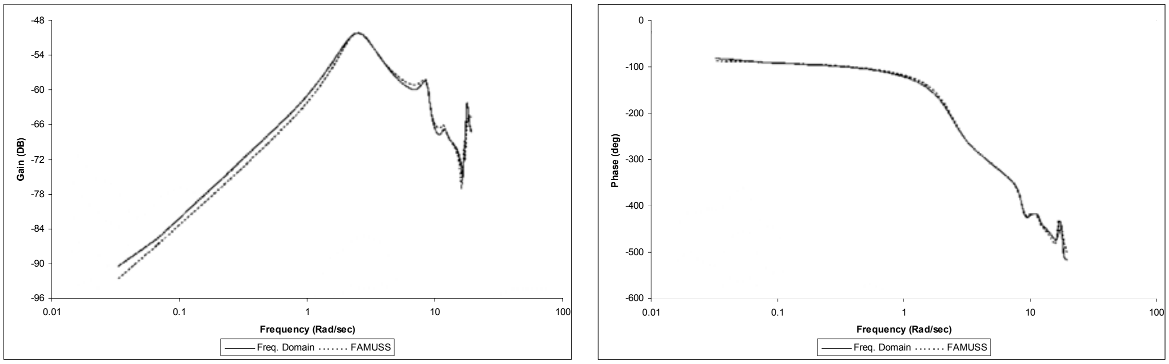

Figure 4: Application of the FAMUSS process to compute CG Acceleration Due to Gust Excitation.

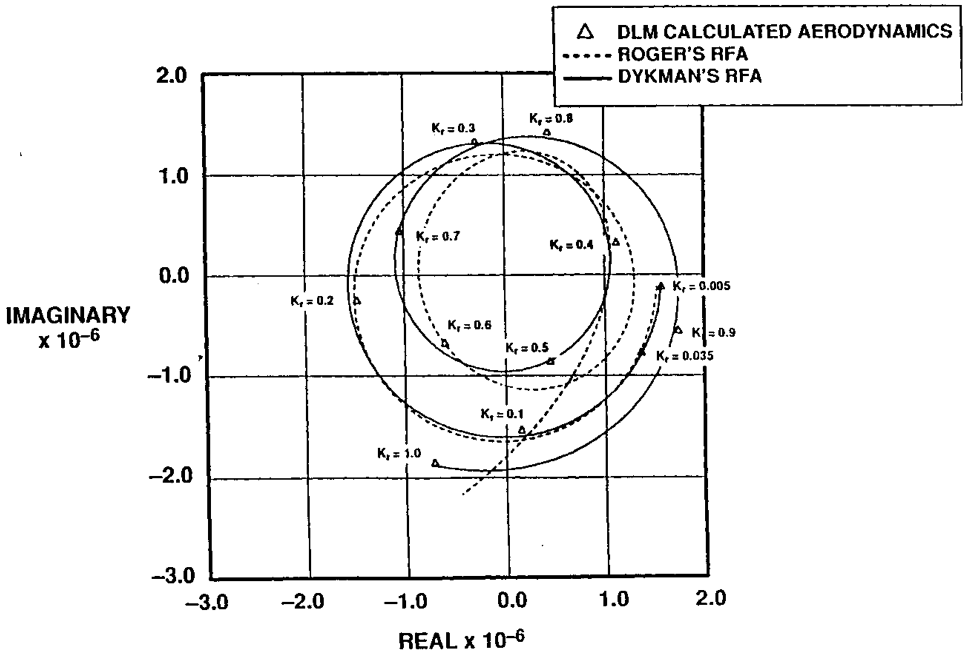

Figure 5: Typical Gust Force Representation.

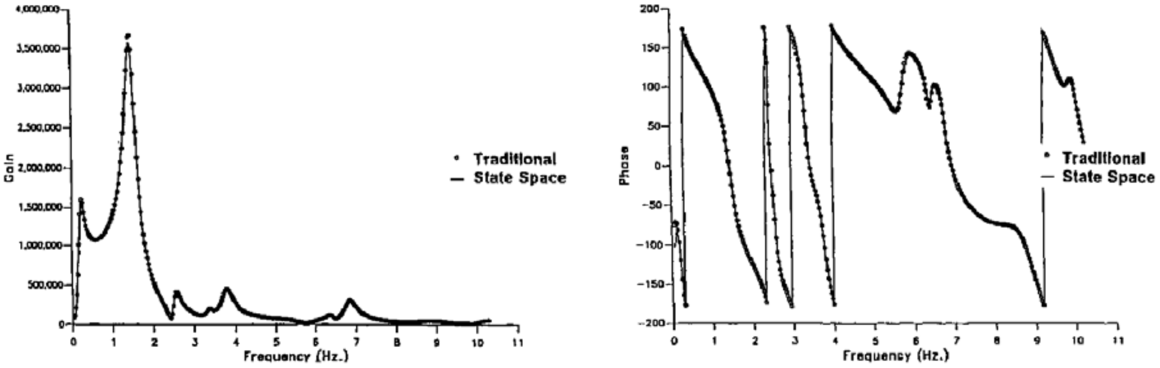

Figure 6: Wing Bending Moment Response Due to Gust Excitation (RFA).

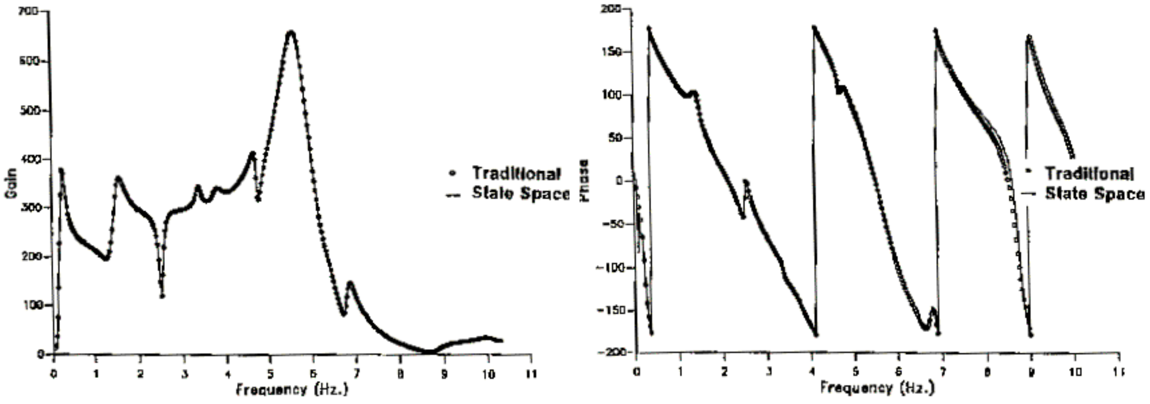

Figure 7: Horizontal Tail Shear Due to Gust Excitation (RFA).

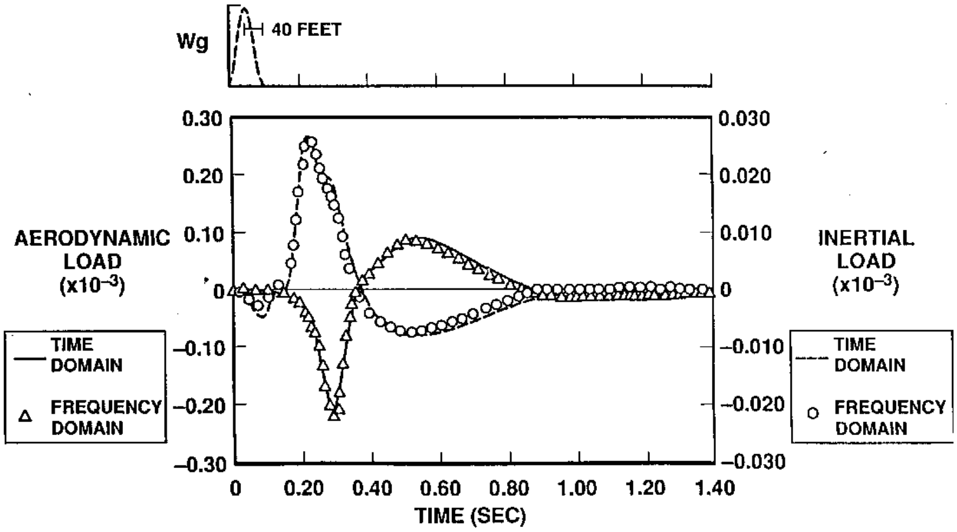

Figure 8: Time-Domain Response: Horizontal Tail Shear Due to 1-Cos Gust Input.

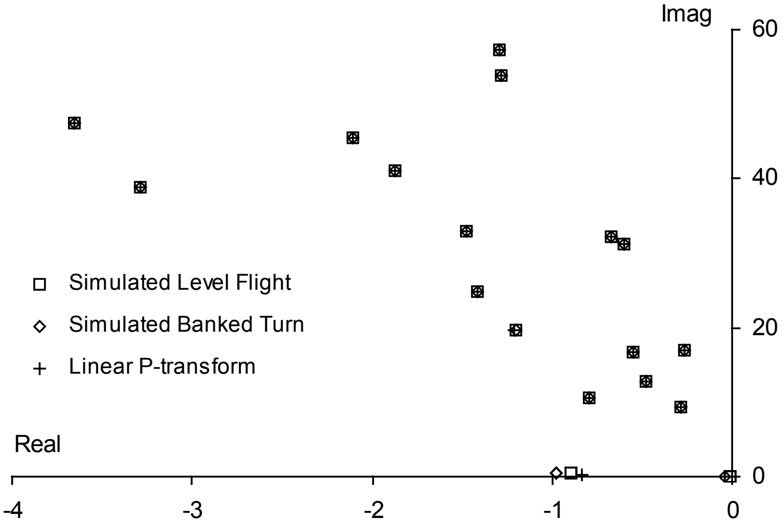

Figure 9: Root Locations for Linear P-transform Analysis and Two Simulated Conditions.

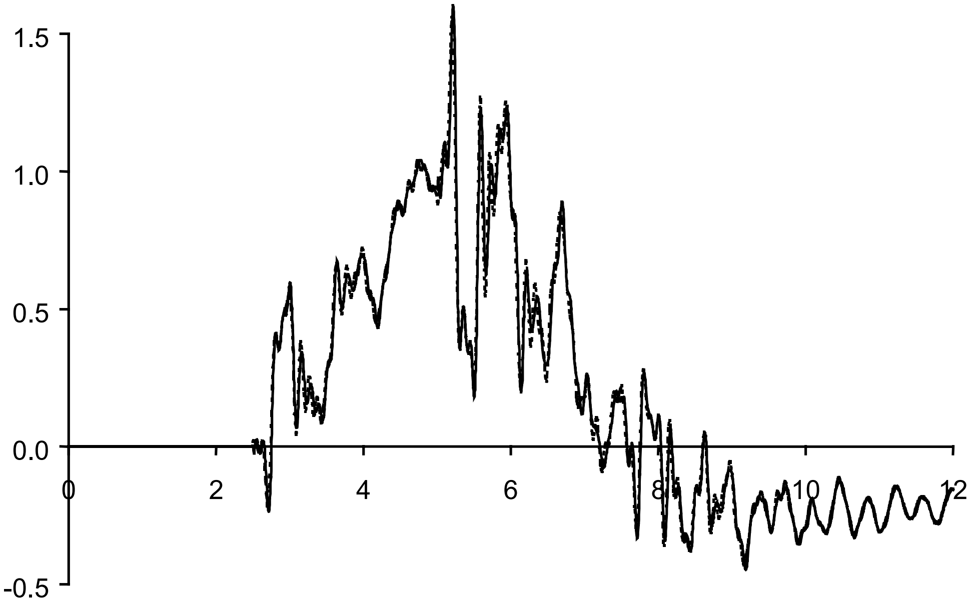

Figure 10: Comparison of Vertical Load Factor Versus Time (sec) at Pilot Station due to Pitch Maneuver. Solid line = P-transform, Dotted line = RFA technique.

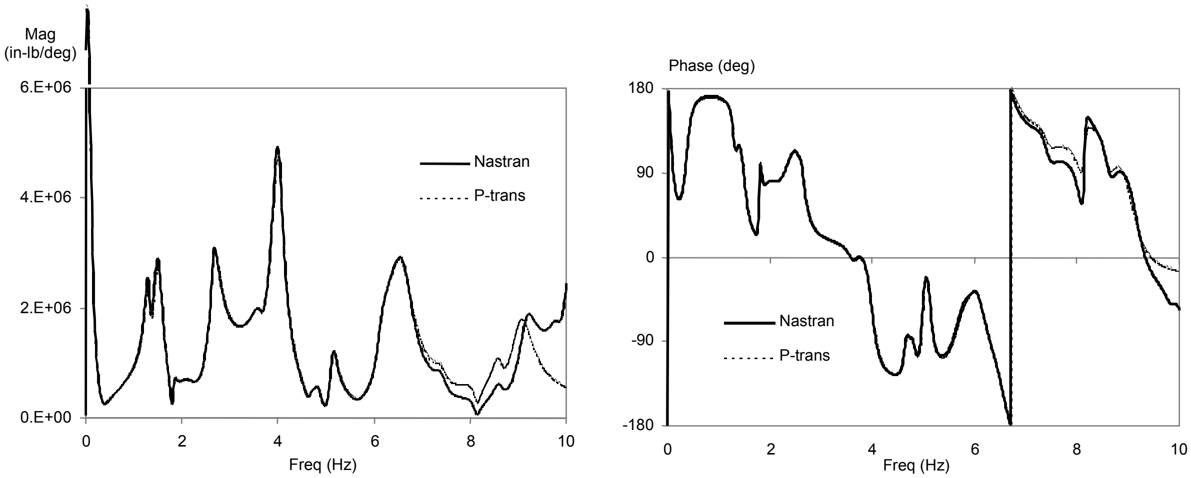

Figure 11: Magnitude and Phase of Wing Bending Moment due to Elevator Excitation